THE RIVERS SECONDARY COLLEGE
The heart of secondary education for Lismore

Assessment Task Notification

RICHMOND RIVER HIGH CAMPUS

Task Number	1	**Task Name**	Reference Notes Test
Course	Mathematics Extension 1	**Faculty**	Mathematics
Teacher	Mr Prince	**Head Teacher**	Mrs Humphrys
Issue date	Term 1, Week 7	**Due date**	Monday 07.04.25 (In Class)
Focus (Topic)	Calculus	**Task Weighting**	20%

Outcomes

ME12-1	A student applies techniques involving proof or calculus to model and solve problems
ME12-4	A student uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution
ME12-6	A student chooses and uses appropriate technology to solve problems in a range of contexts
ME12-7	A student evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

Task description

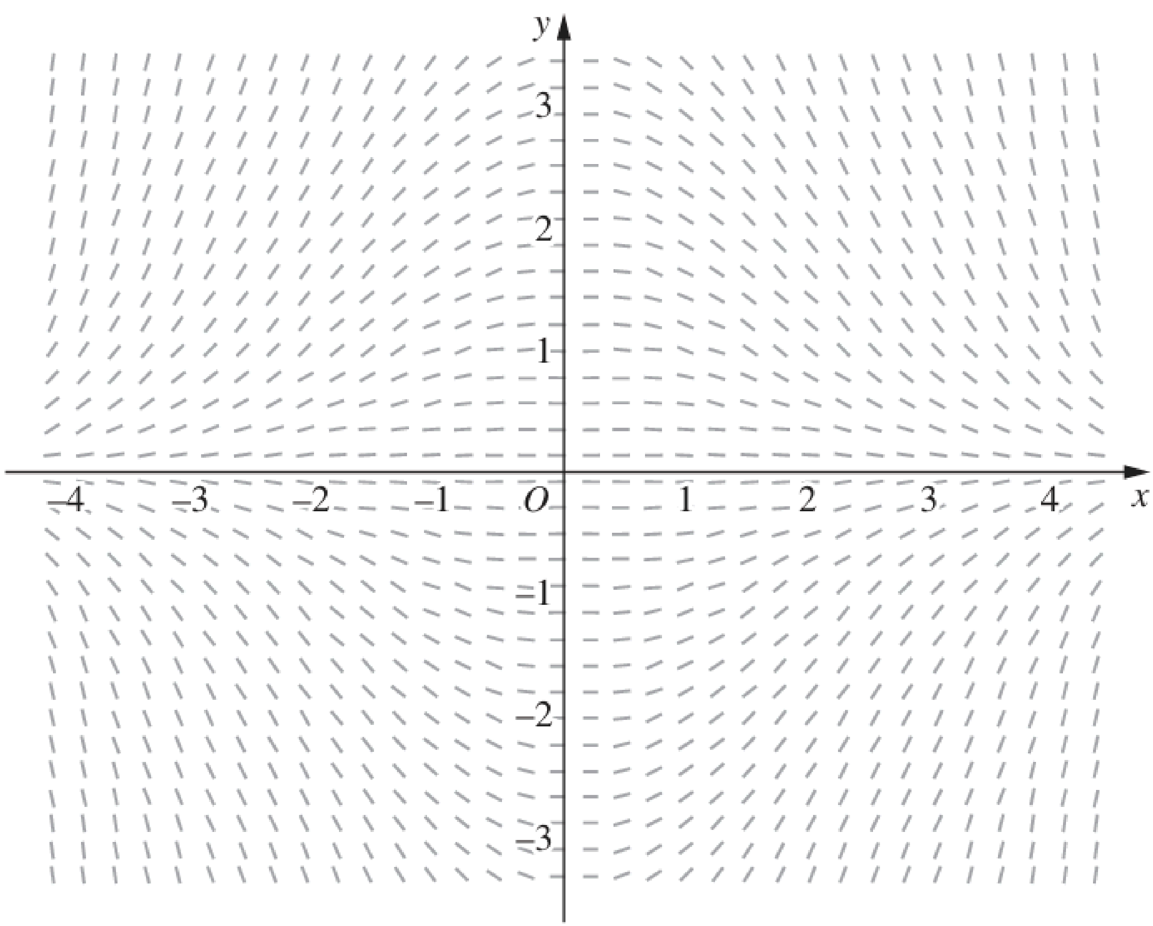
Reference Notes Test - any hand written reference in the students own hand may be used in the exam.
Multiple choice questions. Extended response questions require fully working to be shown.

Marking Guidelines

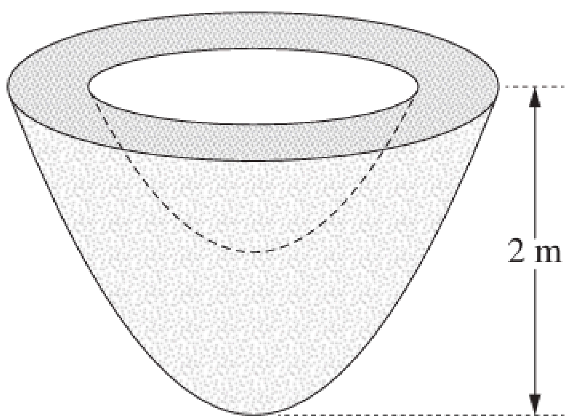
ASSESSMENT CRITERIA: Students will be assessed on their ability to create fully worked solutions using the notation and methodology referred to during the course. See attached PRACTICE TEST - Use the links to explore content and worked solutions

Outcomes from our program:

C3.1a Calculating area of regions between curves determined by functions.
C3.1b Sketching, with and without the use of technology, the graph of a solid of revolution whose boundary is formed by rotating an arc of a function about the x-axis or y-axis.
Calculating the volume of a solid of revolution formed by rotating a region in the plane about the x-axis or y-axis, with and without the use of technology.
Determining the volumes of solids of revolution that are formed by rotating the region between two curves about either the x-axis or y-axis in both real-life and abstract contexts
C3.2a Recognising that an equation involving a derivative is called a differential equation.
Recognising that solutions to differential equations are functions and that these solutions may not be unique.
C3.2b Sketching the graph of a particular solution given a direction field and initial conditions.
Forming a direction field (slope field) from simple first-order differential equations.
Recognising the shape of a direction field from several alternatives given the form of a differential equation, and vice versa.
C3.2d Modelling and solving differential equations (including but not limited to the logistic equation) that arise in situations where rates are involved, for example in chemistry, biology and economics

1.	The slope field for a first order DE is shown.  Which of the following could be the DE represented?	1

	A. $\frac{dy}{dx} = \frac{x}{3y}$ B. $\frac{dy}{dx} = -\frac{x}{3y}$ C. $\frac{dy}{dx} = \frac{xy}{3}$ D. $\frac{dy}{dx} = -\frac{xy}{3}$ 2020 Sample HSC 5 1 ME-C3 Applications of Calculus E2-E3 ME12-4	

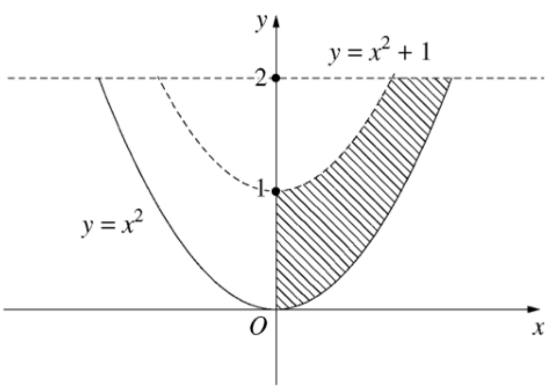
2.	A 2-metre-high sculpture is to be made out of concrete. The sculpture is formed by rotating the region between $y = x^2$, $y = x^2 + 1$ and $y = 2$ around the y-axis.  Find the volume of concrete needed to make the sculpture.	4

2021 Ext 1 HSC
13 (a)Marks 3ME C3 Applications of calculus

Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Evaluates one volume OR • Obtains a correct expression for the volume as a difference of two integrals that are only in terms of y	2
• Obtains an integral for one relevant volume OR • Writes the required volume as the difference of two relevant volumes OR • Finds the limits of integration for both volumes	1

Sample answer:



$$\text{Outer volume} = \pi \int_0^2 x^2\, dy$$

$$= \pi \int_0^2 y\, dy \quad \text{since } y = x^2$$

$$\text{Inner volume} = \pi \int_1^2 x^2\, dy$$

$$= \pi \int_1^2 y - 1\, dy \quad \text{since } y = x^2 + 1$$

The volume of the sculpture is $\frac{3\pi}{2}$ m^3.

The volume of the sculpture is the difference between the outer volume and the inner volume:

$$\therefore V = \int_0^2 \pi y \, dy - \int_1^2 \pi (y-1) \, dy$$

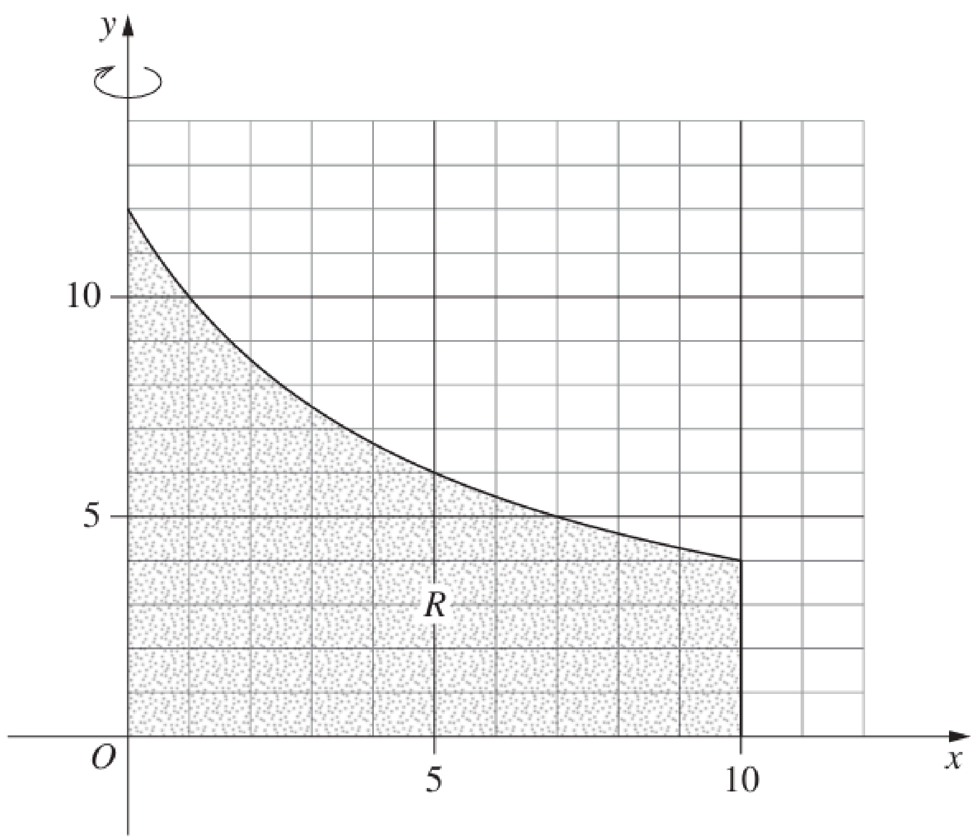
$$V = \pi \left[\frac{y^2}{2} \right]_0^2 - \pi \left[\frac{y^2}{2} - y \right]_1^2$$

$$= \pi \left(\frac{4}{2} - 0 \right) - \pi \left(\left[\frac{4}{2} - 2 \right] - \left[\frac{1}{2} - 1 \right] \right)$$

$$= \frac{3\pi}{2}$$

3.

The region, R, bounded by the hyperbola $y = \dfrac{60}{x+5}$, the line $x = 10$ and the coordinate axes is shown.



Find the volume of the solid of revolution formed when the region R is rotated about the y-axis. Leave your answer in exact form.

4

2023 Ext1 HSC
12 (e) 4 ME-C3 Applications of Calculus ME12-4

Question 12 (e)

Criteria	Marks
• Provides correct solution	4
• Obtains the volume formed by revolving the hyperbola about the *y*-axis, or equivalent merit	3
• Obtains correct integral expression for the volume formed by revolving the hyperbola about the *y*-axis, or equivalent merit	2
• Writes *x* in terms of y, or recognises that the volume is the sum of two simpler volumes, or equivalent merit	1

Sample answer:

The total volume is the sum of the volume when the given hyperbola is revolved about the y-axis, V_1, and the volume of a cylinder, V_2.

$$y = \frac{60}{x+5}$$

$$x + 5 = \frac{60}{y}$$

$$x = \frac{60}{y} - 5$$

$$V_1 = \pi \int_4^{12} x^2 \, dy$$

$$= \pi \int_4^{12} \left(\frac{60}{y} - 5 \right)^2 dy$$

$$= \pi \int_4^{12} \frac{3600}{y^2} - \frac{600}{y} + 25 \, dy$$

$$= \pi \left[-\frac{3600}{y} - 600 \ln y + 25y \right]_4^{12}$$

$$= \pi \left[\left(-\frac{3600}{12} - 600 \ln 12 + 25 \times 12 \right) - \left(-\frac{3600}{4} - 600 \ln 4 + 25 \times 4 \right) \right]$$

$$= \pi (-600 \ln 12 + 800 + 600 \ln 4)$$

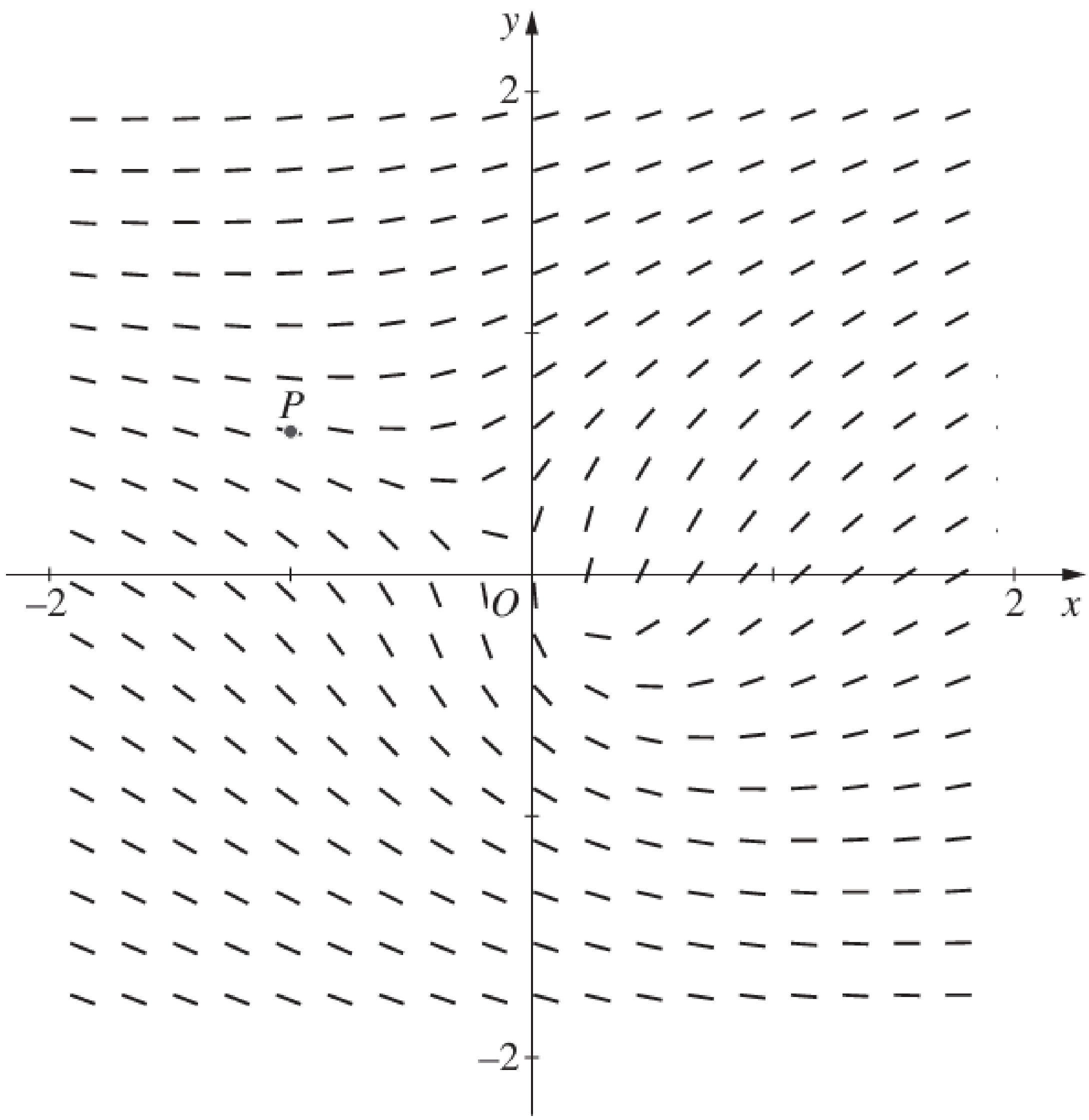
$$= \pi (800 - 600 \ln 3)$$

$$V_2 = \pi \times 10^2 \times 4$$

$$= 400\pi$$

$$\text{Total} = V_1 + V_2$$

$$\therefore \ \text{Total} = \pi(1200 - 600 \ln 3) \ \text{units}^3$$

4.	The direction field for a particular DE is given. The graph of a particular solution to the DE passes through the point P. On the diagram below, sketch the graph of this particular solution 	1 2

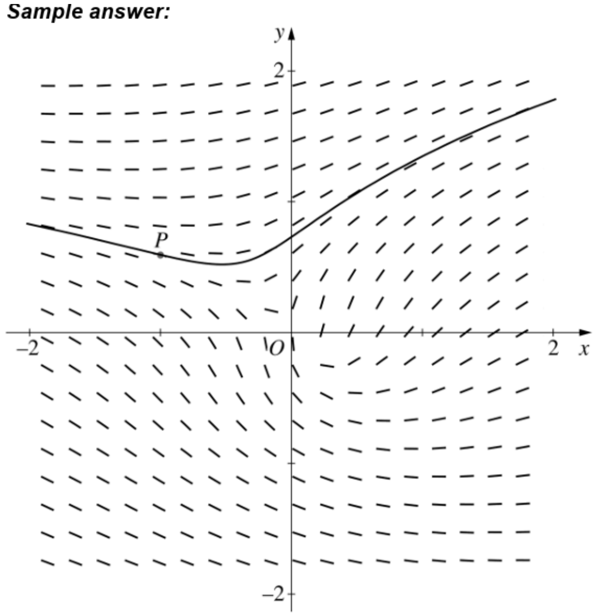
[2021 Ext 1 HSC]

12 (a) 1 ME C3 Applications of calculus ME 12–4

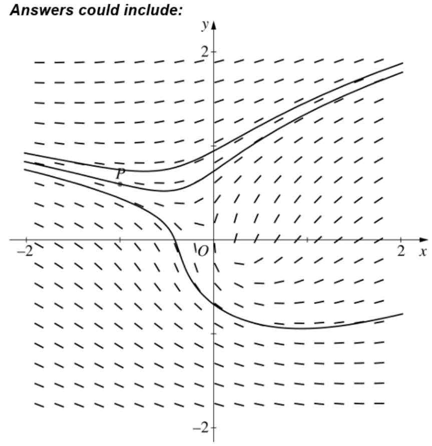
Question 12 (a)

Criteria	Marks
• Provides correct sketch	1

Sample answer:



Answers could include:



Note: An acceptable solution curve does not cross any tangent line in the direction field.

5. A bottle of water, with temperature 5°C, is placed on a table in a room. The temperature of the room remains constant at 25°C. After t minutes, the temperature of the water, in degrees Celsius, is T.

The temperature of the water can be modelled using the differential equation

$$\frac{dT}{dt} = k(T - 25) \qquad \text{(Do NOT prove this.)}$$

where k is the growth constant.

(i) After 8 minutes, the temperature of the water is 10°C.

By solving the differential equation, find the value of t when the temperature of the water reaches 20°C. Give your answer to the nearest minute. 3

(ii) Sketch the graph of T as a function of t. 1

2021 Ext 1 HSC
12 (b) (i) 3 ME C1 Rates of change ME 11-4, ME 12–4
12 (b) (ii) 1 ME C1 Rates of change ME 11–4

Question 12 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Finds the value of k, or equivalent merit	2
• Obtains a solution to the differential equation, that is, $T = 25 + Ae^{kt}$, or equivalent merit OR • Finds the value of A	1

Sample answer:

$$\int \frac{dT}{T-25} = \int k\ dt$$

$$\therefore kt = \ln(T-25) + c$$

$$\therefore T - 25 = Ae^{kt}$$

when $t = 0,\ T = 5$

$$\therefore -20 = A$$

$$\therefore T = 25 - 20e^{kt}$$

when $t = 8,\ T = 10$

$$\therefore 10 = 25 - 20e^{8k}$$

$$-15 = -20e^{8k}$$

$$e^{8k} = \frac{3}{4}$$

$$8k = \ln\left(\frac{3}{4}\right)$$

$$k = \frac{1}{8}\ln\left(\frac{3}{4}\right)$$

when $T = 20$

$$20 = 25 - 20e^{\frac{1}{8}\ln\left(\frac{3}{4}\right)t}$$

$$-5 = -20e^{\frac{1}{8}\ln\left(\frac{3}{4}\right)t}$$

$$\frac{1}{4} = e^{\frac{1}{8}\ln\left(\frac{3}{4}\right)t}$$

$$\frac{1}{8}\ln\left(\frac{3}{4}\right)t = \ln\left(\frac{1}{4}\right)$$

$$\therefore t = \frac{8\ln\left(\frac{1}{4}\right)}{\ln\left(\frac{3}{4}\right)}$$

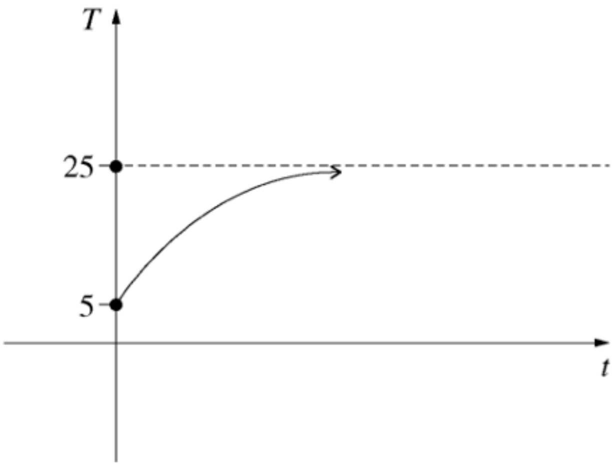
$$= 38.55\ldots$$

$$\simeq 39 \text{ minutes.}$$

Question 12 (b) (ii)

Criteria	Marks
• Provides correct sketch	1

Sample answer:



6.	In a certain country, the population of deer was estimated in 1980 to be 150 000. The population growth is given by the logistic equation $\frac{dP}{dt} = 0.1P\left(\frac{C-P}{C}\right)$ where t is the number of years after 1980 and C is the carrying capacity. In the year 2000, the population of deer was estimated to be 600 000. Use the fact that $\frac{C}{P(C-P)} = \frac{1}{P} + \frac{1}{C-P}$ to show that the carrying capacity is approximately 1 130 000.	4

2021 Ext 1 HSC
14 (b) 3 ME C3 Applications of calculus

Question 14 (b)

Criteria	Marks
• Provides correct solution	4
• Uses the given information to obtain two equations in A and C, or equivalent merit	3
• Integrates both sides correctly, or equivalent merit	2
• Attempts to separate the variables in the differential equation, or equivalent merit	1

Sample answer:

$$\frac{dP}{dt} = 0.1P\left(\frac{C-P}{C}\right)$$

$$\int \frac{C}{P(C-P)}\,dP = \int 0.1\,dt$$

$$\int\left(\frac{1}{P}+\frac{1}{C-P}\right)dP = \int 0.1\,dt$$

$\ln|P| - \ln|C-P| = 0.1t + k$ where k is a constant

$$\ln\left|\frac{P}{C-P}\right| = 0.1t + k$$

$\dfrac{P}{C-P} = Ae^{0.1t}$ where $A = e^k$ is a constant

When $t = 0$, $P = 150\,000$ so $\dfrac{150\,000}{C-150\,000} = A$ ①

When $t = 20$, $P = 600\,000$ so $\dfrac{600\,000}{C-600\,000} = Ae^2$ ②

Substituting ① into ②: $\dfrac{600\,000}{C-600\,000} = \dfrac{150\,000}{C-150\,000}e^2$

Taking the reciprocal of both sides

$$\frac{C-600\,000}{600\,000} = \frac{C-150\,000}{150\,000}e^{-2}$$

$$150\,000(C-600\,000) = 600\,000(C-150\,000)e^{-2}$$

$$C\left(150\,000 - 600\,000e^{-2}\right) = 150\,000\times 600\,000\left(1-e^{-2}\right)$$

$$C = \frac{150\,000\times 600\,000\left(1-e^{-2}\right)}{150\,000-600\,000e^{-2}} \simeq 1\,131\,121$$

$$\simeq 1\,131\,000$$