



Task Number	2	Task Name	Assignment - Proof
Course	Mathematics Extension 2	Faculty	Mathematics
Teacher	Prince	Head Teacher	Humphrys
Issue date	Wed, 6 March 2024	Due date	Thurs, 28.3.24 5pm
Focus (Topic)	Proof	Task Weighting	25%

Outcomes

MEX12-1 - A student understands and uses different representations of numbers and functions to model, prove results and find solutions to problems in a variety of contexts, understands and uses different representations of numbers and functions to model, prove results and find solutions to problems in a variety of contexts.

MEX12-2 - A student chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings.

MEX12-8 - A student communicates and justifies abstract ideas and relationships using appropriate language, notation and logical argument

Task description

5 part investigation requiring students to develop and present a variety of proofs using methods interrogated in this course. Problems to be proven must sit within the scope of the syllabus. Students must be able to locate each proof within the syllabus.

Part 1 - A selection of proof problems.

Part 2 – Students source a complex proof problem that can be solved using at least 2 methods explored in this course.

Part 3 - Students source a complex inequality proof that can be located within the scope of this course's syllabus.

Part 4 – Students make conjectures about the about the geometrical properties of a folded paper manipulative

Part 5 – Students present one proof from either Part 2 or Part 3 **as nominated by the teacher on the day the assignment is due during the lesson.**

Marking Guidelines

See Assessment task

Part 1 (20 marks)

Provide fully worked solution to the 10 questions provided below.

For each question provide a fully annotated version of your solution, details may include the interpretation of the mathematical metalanguage, efficiency, alternate solutions and identifying how this question fits into the syllabus.

1. Prove by contradiction that the square root of the irrational number m is also irrational.
2. Prove or disprove the following statement is false:
 $\exists n$ such that $3^n + 4^n = 5^n$
3. Prove that $\frac{\sin x}{x} \rightarrow 0$ as $x \rightarrow \infty^+$
4. Let $a, b, x, y > 0$. Prove that $(a^2 + b^2)(x^2 + y^2) \geq (ax + by)^2$
5. If $a, b, c > 0$ then prove $a^3 + b^3 + c^3 \geq a^2b + b^2c + c^2a$

Part 1 Marking Criteria:

Mark	Solution for each question
0	Incomplete or no solution provided.
1	Fully worked and correct solution provided.
2	Fully worked and correct solution provided. Annotated, syllabus relevance stated.
3	Fully worked and correct solution provided. Annotated, syllabus relevance stated. Alternative solution provided.
4	Efficiently worked and correct solution provided. Annotated, syllabus relevance stated. Alternative solution provided, Interpretation of mathematical metalanguage provided.

Part 2 (20 marks)

Find a complex proof problem that can be solved using more than one the following methods: -

- i) Direct proof
- ii) Contradiction
- iii) Contrapositive
- iv) Counterexamples

Provide a detailed solutions for the problem.

Provide detailed annotations for the proofs.

Show alternate methods of solving this proof.

Prepare a presentation between 3 and 7 minutes long to introduce and share your findings.

Part 2: Marking Criteria

A (18 – 20)	Question is complex. (spicy or harder) Question is solved using at least 2 methods. Each solution is efficiently worked. Each solution step is annotated. Presentation has a script provided, includes interpretation of mathematical metalanguage. Explains how the term converse is used and why it is not used to prove a statement. Presentation fits time criteria.
B (14 – 17)	Question is complex. (spicy) Question has been solved by one method and significant progress has been made on a second method. The solution is fully worked. The solution steps are annotated. Presentation has a script provided, includes interpretation of mathematical metalanguage. Presentation fits time criteria.
C (10 – 13)	Question is medium level complexity. One solution is fully worked. One has significant detail of progression to a second solution. Each solution step is annotated. Presentation has a script provided, includes interpretation of language. Presentation fits time criteria.
D (5 – 10)	Question is medium level complexity. Question has been solved by one method only. One solution is fully worked Each solution step is annotated. Presentation has a script provided, includes interpretation of language. Presentation fits time criteria.
E (0 – 4)	Question is simple in level. Question has been solved by one method and significant progress has been made on a second method. One solution is fully worked. Each solution step is annotated. Presentation has a script provided, includes interpretation of language. Presentation fits time criteria.

Part 3 (20 marks)

Find a complex inequality proof and identify it within its position to the syllabus (NONE beyond the syllabus scope). To create complexity the question may build from a simple problem to a complex one where earlier solutions are used to solve later parts.

- i) $a > b$ inequalities
- ii) squaring and inequalities
- iii) triangle inequality
- iv) $AM > GM$

Provide a detailed solutions for the problem.

Provide detailed annotations for the proofs.

Show alternate methods of solving this proof.

Prepare a presentation between 3 and 7 minutes long to introduce and share your findings.

Part 3: Marking Criteria

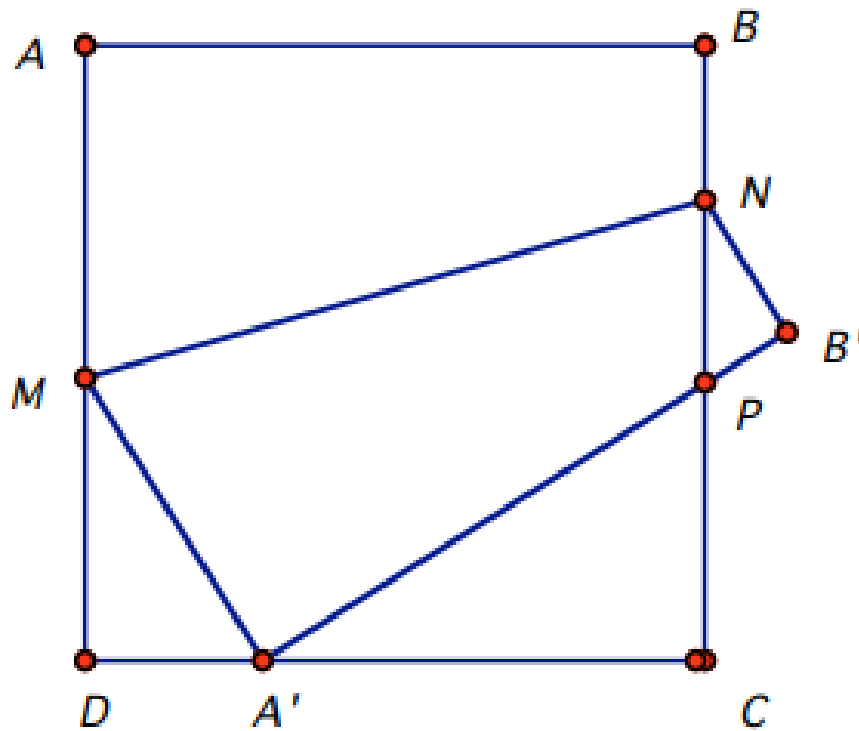
A (18 – 20)	Question is complex. (Spicy or harder) The solution is efficiently worked. Question has links between the syllabus elements, and these are identified. The solution steps are annotated. Presentation has a script provided, includes interpretation of mathematical metalanguage. Presentation fits time criteria.
B (14 – 17)	Question is complex. (spicy) The solution is fully worked. The solution steps are annotated. Presentation has a script provided, includes interpretation of mathematical metalanguage. Presentation fits time criteria.
C (10 – 13)	Question is medium level complexity. The solution is fully worked. The solution steps are annotated. Presentation has a script provided, includes interpretation of mathematical metalanguage. Presentation fits time criteria.
D (5 – 10)	Question is medium level complexity. The solution is fully worked. The solution steps are annotated. Presentation has a script provided.
E (0 – 4)	Question is simple in level. The solution is fully worked. The solution steps are annotated. Presentation has a script provided.

Part 4: Investigation

Part 4: Option 1: Paper folding

Start with a square piece of paper labelled ABCD as shown in the Figure below. Fold the paper so that the point A now lies on segment CD. Move the point A back and forth on the segment CD.

Make conjectures/statements about what you are seeing.



Part 4 : Marking Criteria

A (18 – 20)	<p>Can write at 4 or more conjectures/ statements about the geometrical properties. Can write 4 or more conjectures / statements about the areas formed. Consider the situation where A' lies on the line CD but beyond the limits of the segment CD. Can create 2 implications and write the negation, converse, and contrapositive of this statement. Can analyse one of the implications written above. Can create and analyse the use of inequality/squaring/triangle inequality/AM>GM properties related to this activity.</p>
B (14 – 17)	<p>Can write 3 conjectures/ statements about the geometrical properties. Can write 3 conjectures / statements about the areas formed. Consider one case where A' lies on the line CD but beyond the limits of the segment CD. Can create an implication and write the negation, converse, and contrapositive of this statement. Can analyse one of the implications written above. Can create and analyse the use of inequality/squaring/triangle inequality/AM>GM properties related to this activity.</p>
C (10 – 13)	<p>Can write 2 conjectures/ statements about the geometrical properties. Can write 2 conjectures / statements about the areas formed. Consider one case where A' lies on the line CD but beyond the limits of the segment CD. Can create an implication and write the negation, converse, and contrapositive of this statement. Can make significant steps in the analysis of the implications written above. Can create and analyse the use of inequality/squaring/triangle inequality/AM>GM properties related to this activity.</p>
D (5 – 10)	<p>Can write a conjecture/ statement about the geometrical properties. Can write a conjecture / statement about the areas formed. Consider the properties where A' lies on the line CD but beyond the limits of the segment CD. Can create an implication and write the negation, converse, and contrapositive of this statement. Can make significant steps in the analysis one of the implications written above. Can create an inequality/squaring/triangle inequality/AM>GM using the properties related to this activity.</p>
E (0 – 4)	<p>Made a serious attempt to develop a correct conjectures/ statement about the geometrical properties. Made a serious attempt to develop a correct conjectures / statement about the areas formed. Consider one case where A' lies on the line CD but beyond the limits of the segment CD. Can create an implication and write one of the following that relates to the implication - negation, converse, and contrapositive of this statement.</p>

Part 5 – Presentation (20 marks)

On the day the task has been handed in and considered you will be asked to present either part 2 or part 3 of this assignment.

Teacher will select the presentation.

The presentation should last between 3 and 7 minutes.

Questions may be asked at the end of your presentation.

A (18 – 20)	Discusses why the problem was selected. Identifies where the problem was found. Identifies the successes and struggles they had with this problem. Identifies any other similar problems they found and how these relate to the final presentation. Demonstrates knowledge of the problem during the presentation. Is purposeful and clear with their ideas, explanations, and conclusions. Interested and enthusiastic about the learning. Presentation lasted more than 5 minutes.
B (14 – 17)	Discusses why the problem was selected. Identifies where the problem was found. Identifies a successful step in this learning process. Identifies how they overcame their struggles with this problem. Identifies another similar problem they found and how this related to the final presentation. Demonstrates knowledge of the problem during the presentation. Is purposeful and clear with their ideas and explanations. Interested and enthusiastic about the learning. Presentation was more than 3 minutes
C (10 – 13)	Discusses why the problem was selected. Identifies where the problem was found. Identifies struggles they encountered. Identifies a similar problem and how these helped them complete the task. Demonstrates knowledge of the problem during the presentation. Is purposeful and clear with their ideas. Presentation was more than 3 minutes
D (5 – 10)	Discusses why the problem was selected. Identifies where the problem was found. Identifies struggles they encountered. Identifies a similar problem and how these helped them complete the task. Demonstrates some knowledge of the problem during the presentation. Is mostly purposeful and clear with their ideas. Presentation is close to 3 minutes.
E (0 – 4)	Discusses why the problem was selected. Identifies where the problem was found. Identifies struggles they encountered. Presentation was significantly less than 3 minutes.



Illness/Misadventure Appeal - Application Form

Students may lodge an illness/misadventure appeal application if they believe that circumstances occurring immediately before or during an assessment task and which were beyond their control, diminished their performance, lead to their non-attendance or a late submission of an assessment task. Applications may be in respect of:

- illness or injury – that is, illness or physical injuries suffered directly by the student which allegedly affected the student's performance in an assessment task (e.g. influenza, an asthma attack, a cut hand)
- misadventure – that is, any other event beyond the student's control which allegedly affected the student's performance in an assessment task (e.g. death of a friend or family member, involvement in a traffic accident, isolation caused by a flood). **NB: the NESAs Illness/Misadventure process is to be used for HSC examinations**

Please complete this form and return to the Head Teacher of the subject.

Student name: _____		Year group: _____	
Subject: _____		Class teacher: _____	
Type of task: _____		Head teacher: _____	
Date of task: _____		Appeal applications for a Shared Curriculum subject must be returned to the Head teacher at the campus where the subject is delivered.	
Nature of application <i>(please circle)</i> :			
Extension-late assessment	Absence from assessment task	Special consideration	
Illness	Misadventure		
Basis of appeal <i>(please circle)</i> :			
Reasons for this application including the date, time and duration of illness or misadventure.			
<i>(continue on separate sheet as required, including all supporting documentation)</i>			
In the event of making an appeal application for multiple assessment tasks, please include details of all tasks in the same time period. Return this form and all documentation and/or medical certificate to your home campus Deputy Principal who will liaise with any host campus where applicable			
_____ Date: _____		_____ Date: _____	
Signature		Parent Signature	

Head teacher comment and recommendation: _____			
		Head teacher signature	Date:
Principal's determination: _____			
		Principal signature	Date:
Appeal Upheld	Appeal Declined		
Outcome discussed with student	Signed: _____	Date: _____	
Outcome recorded in Sentral	Signed: _____	Date: _____	